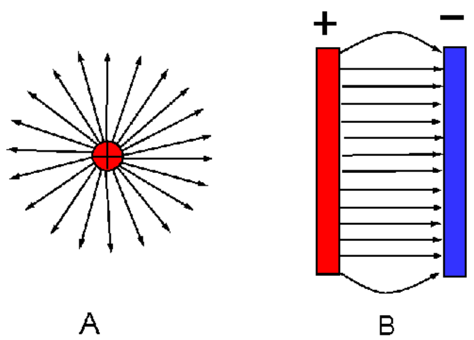
Electric Fields and Circuits

Electric Fields



Electric field is defined as the electric force per unit charge. The direction of the field is taken to be the direction of the force it would exert on a positive test charge.

$$E = \frac{F}{q} \qquad E = \frac{V}{d}$$

Units

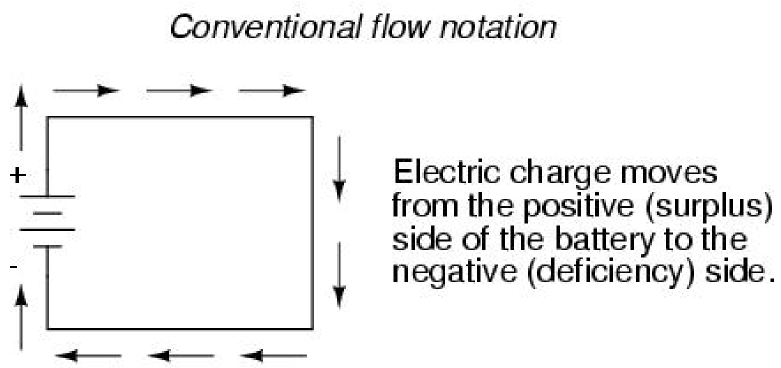
NC^{-1}

Vm^{-1}

Current

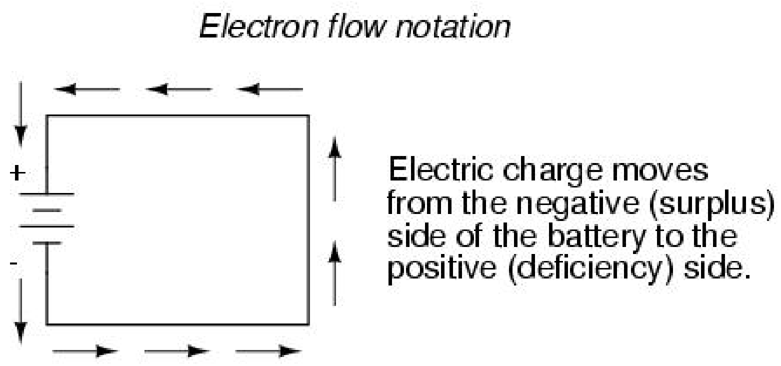
Conventional Current

Direction a positive charge will travel in a field



Electron Flow

Direction an electron will travel in an electric field

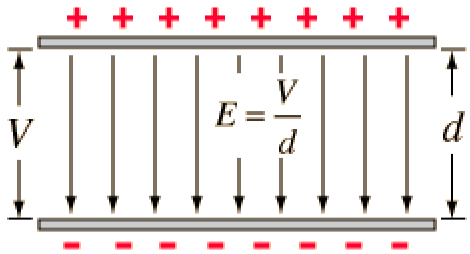


$$I = \frac{q}{t}$$

Units
Amps
C S^{-1}

Voltage
Voltage is electric potential energy per unit charge, measured in joules per coulomb (= volts). It is often referred to as "electric potential", which then must be distinguished from electric potential energy by noting that the "potential" is a "per-unit-charge" quantity.

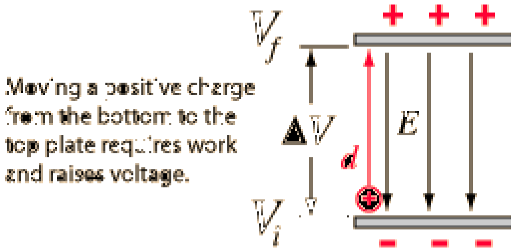
The case of a constant electric field, as between charged parallel plate conductors, is a good example of the relationship between work and voltage.



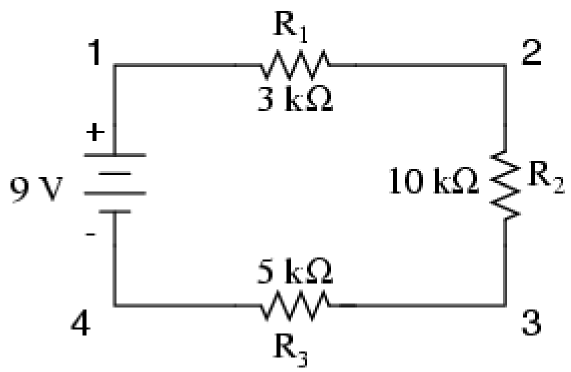
The electric field is by definition the force per unit charge, so that multiplying the field times the plate separation gives the work per unit charge, which is by definition the change in voltage.

The change in voltage is defined as the work done per unit charge against the electric field. In the case of constant electric fieldwhen the movement is directly against the field, this can be written

$$V_f - V_i = \frac{Fd}{q} = -Ed$$

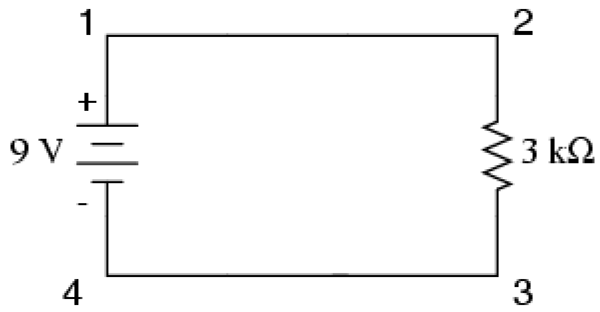


Series Circuits
Let's start with a series circuit consisting of three resistors and a single battery:



The first principle to understand about series circuits is that the amount of current is the same through any component in the circuit. This is because there is only one path for electrons to flow in a series circuit, and because free electrons flow through conductors like marbles in a tube, the rate of flow (marble speed) at any point in the circuit (tube) at any specific point in time must be equal.

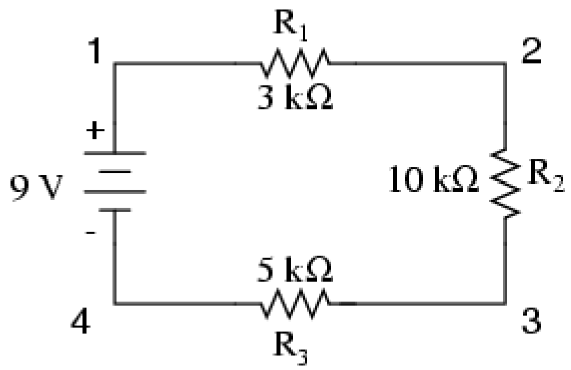
From the way that the 9 volt battery is arranged, we can tell that the electrons in this circuit will flow in a counter-clockwise direction, from point 4 to 3 to 2 to 1 and back to 4. However, we have one source of voltage and three resistances. How do we use Ohm's Law here?
An important caveat to Ohm's Law is that all quantities (voltage, current, resistance, and power) must relate to each other in terms of the same two points in a circuit. For instance, with a single-battery, single-resistor circuit, we could easily calculate any quantity because they all applied to the same two points in the circuit:



$$I = \frac{E}{R}$$

$$I = \frac{9 \text{ volts}}{3 \text{ k}\Omega} = 3 \text{ mA}$$

Since points 1 and 2 are connected together with wire of negligible resistance, as are points 3 and 4, we can say that point 1 is electrically common to point 2, and that point 3 is electrically common to point 4. Since we know we have 9 volts of electromotive force between points 1 and 4 (directly across the battery), and since point 2 is common to point 1 and point 3 common to point 4, we must also have 9 volts between points 2 and 3 (directly across the resistor). Therefore, we can apply Ohm's Law (I = E/R) to the current through the resistor, because we know the voltage (E) across the resistor and the resistance (R) of that resistor. All terms (E, I, R) apply to the same two points in the circuit, to that same resistor, so we can use the Ohm's Law formula with no reservation.
However, in circuits containing more than one resistor, we must be careful in how we apply Ohm's Law. In the three-resistor example circuit below, we know that we have 9 volts between points 1 and 4, which is the amount of electromotive force trying to push electrons through the series combination of R_1, R_2, and R_3. However, we cannot take the value of 9 volts and divide it by 3k, 10k or 5k Ω to try to find a current value, because we don't know how much voltage is across any one of those resistors, individually.



The figure of 9 volts is a *total* quantity for the whole circuit, whereas the figures of 3k, 10k, and 5k Ω are *individual* quantities for individual resistors. If we were to plug a figure for total voltage into an

Ohm's Law equation with a figure for individual resistance, the result would not relate accurately to any quantity in the real circuit.
For R_1, Ohm's Law will relate the amount of voltage across R_1 with the current through R_1, given R_1's resistance, 3kΩ:

$$I_{R1} = \frac{E_{R1}}{3\ \text{k}\Omega} \qquad E_{R1} = I_{R1}(3\ \text{k}\Omega)$$

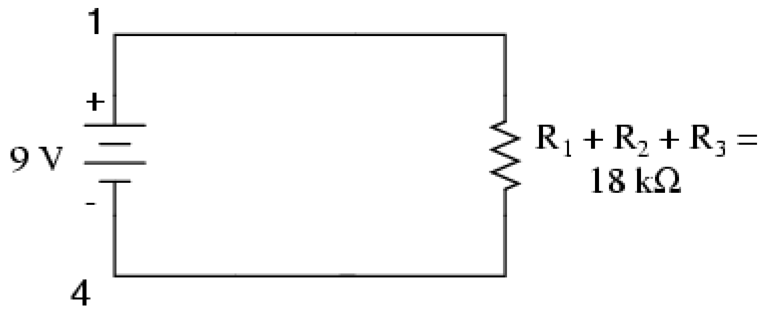
But, since we don't know the voltage across R_1 (only the total voltage supplied by the battery across the three-resistor series combination) and we don't know the current through R_1, we can't do any calculations with either formula. The same goes for R_2 and R_3: we can apply the Ohm's Law equations if and only if all terms are representative of their respective quantities between the same two points in the circuit.
So what can we do? We know the voltage of the source (9 volts) applied across the series combination of R_1, R_2, and R_3, and we know the resistances of each resistor, but since those quantities aren't in the same context, we can't use Ohm's Law to determine the circuit current. If only we knew what the *total* resistance was for the circuit: then we could calculate *total* current with our figure for *total* voltage (I=E/R).
This brings us to the second principle of series circuits: the total resistance of any series circuit is equal to the sum of the individual resistances. This should make intuitive sense: the more resistors in series that the electrons must flow through, the more difficult it will be for those electrons to flow. In the example problem, we had a 3 kΩ, 10 kΩ, and 5 kΩ resistor in series, giving us a total resistance of 18 kΩ:

$$R_{total} = R_1 + R_2 + R_3$$

$$R_{total} = 3\ \text{k}\Omega + 10\ \text{k}\Omega + 5\ \text{k}\Omega$$

$$R_{total} = 18\ \text{k}\Omega$$

In essence, we've calculated the equivalent resistance of R_1, R_2, and R_3 combined. Knowing this, we could re-draw the circuit with a single equivalent resistor representing the series combination of R_1, R_2, and R_3:

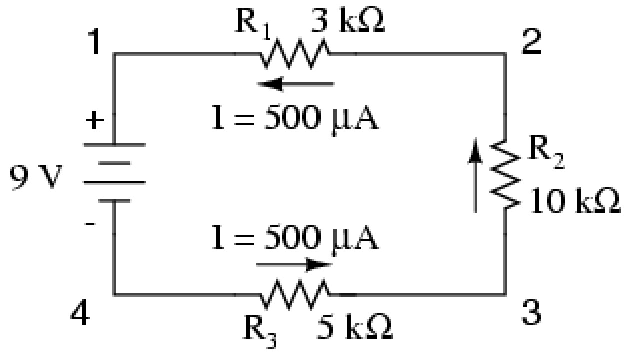


Now we have all the necessary information to calculate circuit current, because we have the voltage between points 1 and 4 (9 volts) and the resistance between points 1 and 4 (18 kΩ):

$$I_{total} = \frac{E_{total}}{R_{total}}$$

$$I_{total} = \frac{9\ \text{volts}}{18\ \text{k}\Omega} = 500\ \mu\text{A}$$

Knowing that current is equal through all components of a series circuit (and we just determined the current through the battery), we can go back to our original circuit schematic and note the current through each component:



Now that we know the amount of current through each resistor, we can use Ohm's Law to determine the voltage drop across each one (applying Ohm's Law in its proper context):

$$E_{R1} = I_{R1}\, R_1 \qquad E_{R2} = I_{R2}\, R_2 \qquad E_{R3} = I_{R3}\, R_3$$

$$E_{R1} = (500\ \mu A)(3\ k\Omega) = 1.5\ V$$

$$E_{R2} = (500\ \mu A)(10\ k\Omega) = 5\ V$$

$$E_{R3} = (500\ \mu A)(5\ k\Omega) = 2.5\ V$$

Notice the voltage drops across each resistor, and how the sum of the voltage drops (1.5 + 5 + 2.5) is equal to the battery (supply) voltage: 9 volts. This is the third principle of series circuits: that the supply voltage is equal to the sum of the individual voltage drops.

However, the method we just used to analyze this simple series circuit can be streamlined for better understanding. By using a table to list all voltages, currents, and resistances in the circuit, it becomes very easy to see which of those quantities can be properly related in any Ohm's Law equation:

	R_1	R_2	R_3	Total	
E					Volts
I					Amps
R					Ohms
	↑ Ohm's Law	↑ Ohm's Law	↑ Ohm's Law	↑ Ohm's Law	

The rule with such a table is to apply Ohm's Law only to the values within each vertical column. For instance, E_{R1} only with I_{R1} and R_1; E_{R2} only with I_{R2} and R_2; etc. You begin your analysis by filling in those elements of the table that are given to you from the beginning:

	R_1	R_2	R_3	Total	
E				**9**	Volts
I					Amps
R	**3k**	**10k**	**5k**		Ohms

As you can see from the arrangement of the data, we can't apply the 9 volts of E_T (total voltage) to any of the resistances (R_1, R_2, or R_3) in any Ohm's Law formula because they're in different columns. The 9 volts of battery voltage is *not* applied directly across R_1, R_2, or R_3. However, we can use our "rules" of series circuits to fill in blank spots on a horizontal row. In this case, we can use the series rule of resistances to determine a total resistance from the *sum* of individual resistances:

	R_1	R_2	R_3	Total	
E				9	Volts
I					Amps
R	3k	10k	5k	**18k**	Ohms

Rule of series circuits

$$R_T = R_1 + R_2 + R_3$$

Now, with a value for total resistance inserted into the rightmost ("Total") column, we can apply Ohm's Law of I=E/R to total voltage and total resistance to arrive at a total current of 500 µA:

	R_1	R_2	R_3	Total	
E				9	Volts
I				**500µ**	Amps
R	3k	10k	5k	18k	Ohms

Ohm's Law

Then, knowing that the current is shared equally by all components of a series circuit (another "rule" of series circuits), we can fill in the currents for each resistor from the current figure just calculated:

	R_1	R_2	R_3	Total	
E				9	Volts
I	**500µ**	**500µ**	**500µ**	**500µ**	Amps
R	3k	10k	5k	18k	Ohms

Rule of series circuits

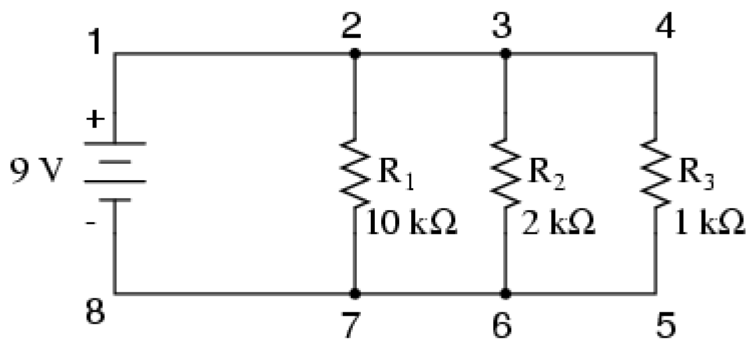
$$I_T = I_1 = I_2 = I_3$$

Finally, we can use Ohm's Law to determine the voltage drop across each resistor, one column at a time:

	R_1	R_2	R_3	Total	
E	**1.5**	**5**	**2.5**	9	Volts
I	500µ	500µ	500µ	500µ	Amps
R	3k	10k	5k	18k	Ohms
	Ohm's Law	*Ohm's Law*	*Ohm's Law*		

Parallel Circuits

Let's start with a parallel circuit consisting of three resistors and a single battery:



The first principle to understand about parallel circuits is that the voltage is equal across all components in the circuit. This is because there are only two sets of electrically common points in a parallel circuit, and voltage measured between sets of common points must always be the same at any given time. Therefore, in the above circuit, the voltage across R_1 is equal to the voltage across R_2 which is equal to the voltage across R_3 which is equal to the voltage across the battery. This equality of voltages can be represented in another table for our starting values:

	R_1	R_2	R_3	Total	
E	9	9	9	9	Volts
I					Amps
R	10k	2k	1k		Ohms

Just as in the case of series circuits, the same caveat for Ohm's Law applies: values for voltage, current, and resistance must be in the same context in order for the calculations to work correctly. However, in the above example circuit, we can immediately apply Ohm's Law to each resistor to find its current because we know the voltage across each resistor (9 volts) and the resistance of each resistor:

$$I_{R1} = \frac{E_{R1}}{R_1} \qquad I_{R2} = \frac{E_{R2}}{R_2} \qquad I_{R3} = \frac{E_{R3}}{R_3}$$

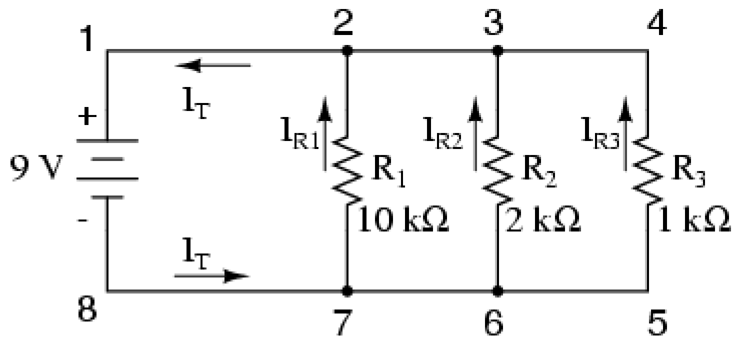
$$I_{R1} = \frac{9\ \text{V}}{10\ \text{k}\Omega} = 0.9\ \text{mA}$$

$$I_{R2} = \frac{9\ \text{V}}{2\ \text{k}\Omega} = 4.5\ \text{mA}$$

$$I_{R3} = \frac{9\ \text{V}}{1\ \text{k}\Omega} = 9\ \text{mA}$$

	R_1	R_2	R_3	Total	
E	9	9	9	9	Volts
I	**0.9m**	**4.5m**	**9m**		Amps
R	10k	2k	1k		Ohms
	↑ *Ohm's Law*	↑ *Ohm's Law*	↑ *Ohm's Law*		

At this point we still don't know what the total current or total resistance for this parallel circuit is, so we can't apply Ohm's Law to the rightmost ("Total") column. However, if we think carefully about what is happening it should become apparent that the total current must equal the sum of all individual resistor ("branch") currents:



As the total current exits the negative (-) battery terminal at point 8 and travels through the circuit, some of the flow splits off at point 7 to go up through R_1, some more splits off at point 6 to go up through R_2, and the remainder goes up through R_3. Like a river branching into several smaller streams, the combined flow rates of all streams must equal the flow rate of the whole river. The same thing is encountered where the currents through R_1, R_2, and R_3 join to flow back to the positive terminal of the battery (+) toward point 1: the flow of electrons from point 2 to point 1 must equal the sum of the (branch) currents through R_1, R_2, and R_3.

This is the second principle of parallel circuits: the total circuit current is equal to the sum of the individual branch currents. Using this principle, we can fill in the I_T spot on our table with the sum of I_{R1}, I_{R2}, and I_{R3}:

	R_1	R_2	R_3	Total	
E	9	9	9	9	Volts
I	0.9m	4.5m	9m	**14.4m**	Amps
R	10k	2k	1k		Ohms

Rule of parallel circuits

$$I_{total} = I_1 + I_2 + I_3$$

Finally, applying Ohm's Law to the rightmost ("Total") column, we can calculate the total circuit resistance:

	R_1	R_2	R_3	Total	
E	9	9	9	9	Volts
I	0.9m	4.5m	9m	14.4m	Amps
R	10k	2k	1k	**625**	Ohms

$$R_{total} = \frac{E_{total}}{I_{total}} = \frac{9\ \text{V}}{14.4\ \text{mA}} = 625\ \Omega$$

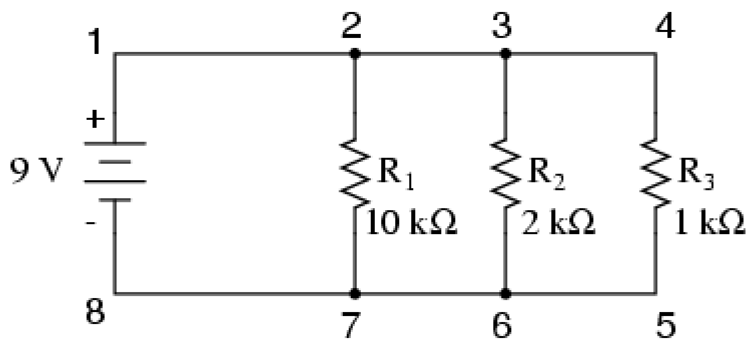
Ohm's Law

Please note something very important here. The total circuit resistance is only 625 Ω: *less* than any one of the individual resistors. In the series circuit, where the total resistance was the sum of the individual resistances, the total was bound to be *greater* than any one of the resistors individually. Here in the parallel circuit, however, the opposite is true: we say that the individual resistances *diminish* rather than *add* to make the total. This principle completes our triad of "rules" for parallel circuits, just as series circuits were found to have three rules for voltage, current, and resistance. Mathematically, the relationship between total resistance and individual resistances in a parallel circuit looks like this:

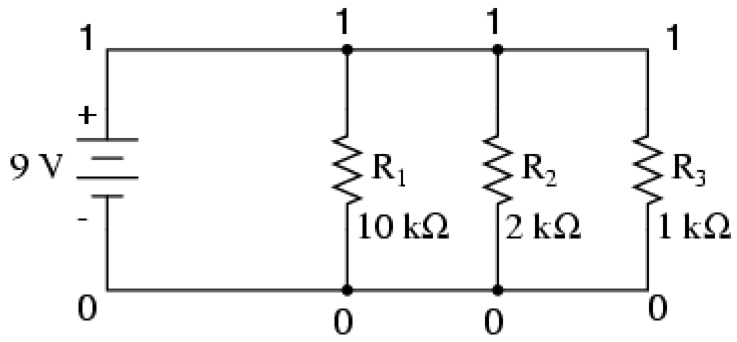
$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

The same basic form of equation works for *any* number of resistors connected together in parallel, just add as many 1/R terms on the denominator of the fraction as needed to accommodate all parallel resistors in the circuit.

Just as with the series circuit, we can use computer analysis to double-check our calculations. First, of course, we have to describe our example circuit to the computer in terms it can understand. I'll start by re-drawing the circuit:



Once again we find that the original numbering scheme used to identify points in the circuit will have to be altered for the benefit of SPICE. In SPICE, all electrically common points must share identical node numbers. This is how SPICE knows what's connected to what, and how. In a simple parallel circuit, all points are electrically common in one of two sets of points. For our example circuit, the wire connecting the tops of all the components will have one node number and the wire connecting the bottoms of the components will have the other. Staying true to the convention of including zero as a node number, I choose the numbers 0 and 1:



An example like this makes the rationale of node numbers in SPICE fairly clear to understand. By having all components share common sets of numbers, the computer "knows" they're all connected in parallel with each other.